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By Jonathan Bostic and Tim Jacobbe

A four-day intervention proved highly successful. Here are practical, specific guidelines for you to quickly develop a similar classroom culture.

Fourteen fifth-grade students gather at the front of the classroom as their summer school instructor introduces Jonathan Bostic as the mathematics teacher for the week. Before examining any math problems, Bostic sits at eye level with the students and informs them that they will solve problems over the next four days by working individually as well as collaborating in small groups and as a whole class.

Bostic describes the problems as more involved than those that students typically encounter. He emphasizes the importance of being open to learning from one another: "We are going to listen carefully to one another because we can learn from something that someone shares during discussion." To that end, he describes and models one appropriate and one inappropriate method for sharing solution strategies during small-group and whole-class discussions.

Bostic also solicits feedback from the students about making the classroom a safe, comfortable problem-solving arena. Finally, he stresses that the solution strategy or representation—rather than the answer—will be the intended focus of discussions.

Discourse and problem solving

A supportive environment where students feel comfortable sharing their solution strategies with one another is crucial for developing a positive classroom culture (NRC 2000).

These students expressed the desire to squeeze in many opportunities to learn from each problem, especially if they might see only one problem per day.

Such a culture promotes *problem-solving discourse*, which is student-centered, problem-focused dialogue among small groups or as a whole class. When children discuss their ideas for interpreting and solving a problem during a well-orchestrated discussion, they support one another's learning and provide information about their own mathematical knowledge.

Communication is an essential component for mathematics learning (NCTM 2000). Mathematics discourse should not be an objective to achieve but rather a process for students to practice and employ (Lampert and Cobb 2003). Teachers can provide scaffolding during whole-class and peer-to-peer discussions by negotiating meaning, promoting students' control of their thoughts and actions via a transfer of responsibility for learning, and by providing intrinsic support during mathematics instruction (Turner et al. 1998). For instance, a teacher may furnish a bridge for students from informal to formal mathematical language.

Certainly, teachers may scaffold their instruction by acting as moderator or closing discussant of a whole-class conversation. Students feel a sense of belonging in classrooms where mathematical discussions are prevalent (Turner et al. 1998). Those who are highly involved in their learning will experience more autonomy, enhanced motivation, and greater confidence than learners who are less involved (Turner et al. 1998). Turner's study highlighted an aspect of classroom culture where discussion of students' ideas was a focus. Students in these classrooms embraced learning from both correct and incorrect strategies and solutions when teachers structured their lessons around students' notions. (On the other hand, instruction in elementary school classrooms lacking discourse may emphasize only error-free learning [Turner et al. 1998].) As a result, discourse-rich classrooms are student centered and can promote sense making. These potential benefits support the idea that problem-focused discourse should be a focal point of mathematics instruction.

Background

Before the study recounted in this article, the summer school instructor had placed students in a primarily single-sex seating arrangement at four separate stations of four or six desks. He believed this arrangement limited classroom disruptions and off-task behavior. Most girls were seated at one station near the back of the room; no more than three boys were at any one station. The instructor stated that before the study nearly all the boys routinely misbehaved during mathematics instruction and that the girls engaged in off-task behaviors.

The summer school instructor typically stood at the chalkboard and lectured to students. Students did not routinely discuss mathematics. The instructor generally reinforced students' learning about a single strategy each day. While students completed follow-up practice exercises, he informed them whether their answers were correct. Students' behaviors and language showed their disinterest and lack of enthusiasm regarding mathematics and learning new strategies.

During this study, the regular summer school instructor acted as an observer while Bostic took on the role of teacher. By describing how the classroom culture and discourse changed over the course of a four-day period, the authors aim to offer teachers a method for creating a similar classroom environment. All other names used in this article are pseudonyms.

A problem-solving culture

Hoping to foster content-relevant conversations between students, Bostic led the class in a discussion of whether the current classroom layout was optimal for them to solve problems and discuss strategies. Students suggested that they be allowed to work where they felt comfortable, whether it was at their desks or seated in chairs near the front of the room. Essentially, students wanted the choice of determining where to work. During individual think time, most students stayed seated at desks; some worked in chairs using clipboards; and one student preferred working alone at a desk near the back of the room. From day one forward, students maintained focus and exhibited no off-task behavior during problem-solving activities. Later they described a sense of empowerment in having such options; they perceived themselves as being in control of their learning.

Students also suggested initiating conversations by asking one another, What did you do? or Tell me how you did [the problem]. Students and teacher alike agreed that to benefit from such learning, everyone should feel welcome to share their work and be open to questions and feedback. Bostic encouraged students with examples of giving positive feedback: "I like your idea for solving the problem that way."

If students disagreed with something a classmate said, they were instructed to direct a question to the problem solver and focus on the work rather than criticize the student. Finally, students indicated the necessity of fully exploring each question and their problem-solving strategies to maximize learning. They maintained that squeezing as many opportunities to learn from each problem was prudent, especially if they might see only one problem each day.

Daily problem-solving lessons

At the start of each day during this week of the study, class began with students and teacher convening near the front of the room. Bostic provided a problem on a handout, then students dispersed throughout the room. The daily items were written to highlight contexts relevant to students' interests and experiences. Questions from *Everyday Mathematics* (Bell et al. 2004) and the Investigations in Number, Data, and Space curriculum (TERC 2004) supplied Bostic with a framework for writing new tasks that could be solved using multiple strategies.

Students completed one problem each of the first two days and solved two problems on each subsequent day. Bostic did not verify their answers during the week; students asked one another for verification. Bostic informed students that they would know whether an answer was correct by listening to whether it was adequately justified using mathematical content and reasoning. Each authentic problem could be solved using several strategies, and the problems varied in complexity.

Using a modified think-pair-share format, students examined a problem for approximately 10–15 minutes. Next, they asked nearby peers to explain for another 10–15 minutes how they had solved the problem. Finally, everyone assembled at the front of the room for 20–30 minutes, depending on the problem, to share problem-solving strategies. Students wrote on poster paper rather than the chalkboard to present their solutions to the class. Then they discussed their strategies with their peers. After they felt they had exhausted all possible solution strategies, they moved their posters to another wall, where students could use them as a reference during future problem-solving activities.

Guidelines to support classroom discourse

Making small changes in your classroom layout and instruction can foster big changes toward a culture that promotes discourse. The authors found the following guidelines to be successful in setting up such an environment.

Be a problem solver

Rather than being a human answer key, be an embedded problem solver. Students in this classroom perceived the teacher as a fellow problem solver instead of the mathematical authority. During individual classroom instruction, Bostic worked by himself at a student's desk, seated in a student's chair. The small-group instructional time permitted him an opportunity to join students and become a part of their conversations. He felt able to informally assess students throughout the week by engaging in these conversations. After the first day, students stopped asking him whether their answers were correct and instead justified their solutions to themselves or their peers.

Support students' learning needs

Allow students to work in ways that support their diverse learning needs. Rows of desks promote a teacher-centered classroom and eschew discussions. Consider creating a variety of desk or table arrangements. Ask students how they would prefer to learn mathematics, and let them work in the manner that they suggest. Giving students seating options allows them to perceive the teacher as less authoritarian and more focused on learning.

In this classroom, some students preferred to write on clipboards near the front of the room, whereas one student preferred working by himself away from others. Bostic placed desks in groups of three or four, in pairs, and as single desks away from others. Students were given the option to work where they wanted and were allowed to move desks and chairs. When they chose a preferable work location, they seemed to stay on task rather than procrastinate or create distractions. The summer school instructor confirmed that during individual work time, Rather than an objective to achieve, mathematics discourse is a process for students to practice.

students remained on task more often than they had in prior weeks.

Use relevant problems

Math problems germane to students' experiences link their in- and out-of-school lives with mathematics. Instruct with mathematically rich problems instead of exercises. In many classrooms like this one, high-stakes testing pushes the summer school instructor to focus on applying procedures so that students may complete exercises, which often lack any context. An engaging atmosphere and contextually relevant problems with multiple entry points that can be solved using more than one strategy—all these characteristics make students more likely to master specific strategies while building a conceptual understanding of mathematical topics.

Modify a think-pair-share strategy

Give students an opportunity to think independently. These students used individual time to understand a given problem and consider possible solution strategies. When students work independently, the teacher should act as support during initial thinking without leading them too much. For instance, if students inquire what to do first, ask them what they think they should do and encourage them to carry out their idea. At times, students may need encouragement to try an idea and feel supported even if the strategy does not lead to the correct solution.

Small-group work is important because task-relevant dialogue promotes a key NCTM Process Standard: Communication (NCTM 2000). Students who talk about mathematics with one another may perceive mathematically relevant communication as a means for learning content. By devoting time to small-group problem solving, students begin to value other's ideas.

Whole-class discussion offers each student an opportunity to learn from peers as well as a time for students to share their ideas as the class facilitator. In this classroom, sharing ideas during the whole-class discussion grew—from an overall lack of voluntary participation to day four, when nearly every student raised his or her hand. Discussing ideas with the entire class permitted students a chance to examine and practice employing others' strategies.

Facilitate reflection

As the teacher, synthesize successful strategies. After the class discussion, Bostic encouraged students to reflect internally on the solution strategies that worked for a problem. If students need to learn strategies that were not part of the discussion, bring an idea to their attention: "I saw a former student use this strategy…."

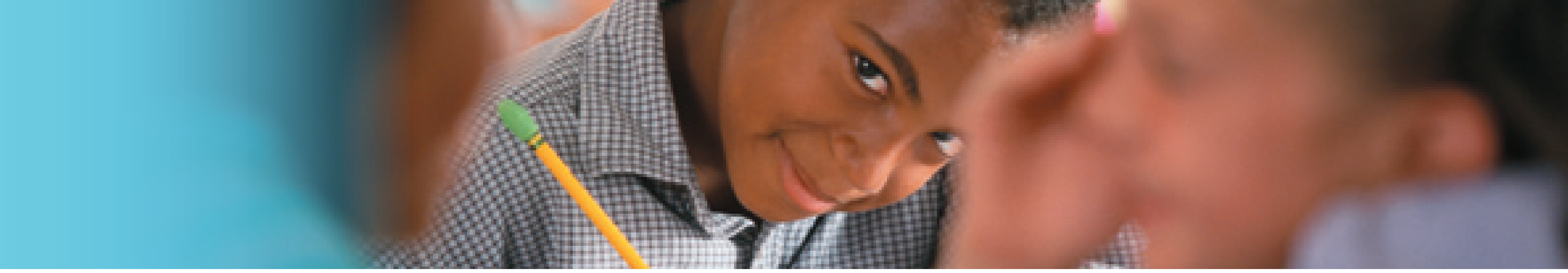
The way you phrase your suggestions is important; cast them so that they seem to come from previous students' work rather than from the teacher. When students perceive a suggestion as coming from the teacher, they may view that strategy as the "best" strategy. Consider rewriting students' solution strategies in a clearer or larger font on poster paper and posting them around the room for students' reference. Students said that seeing their own mathematical strategies posted on the classroom walls was "cool."

Scaffold

While problem solving, some students may need to be reoriented through scaffolding. These students were asked to examine solution strategies that worked for previous problems and determine whether those strategies could lead to success for the stated problem. Many students who indicated they were stuck were able to solve a problem after the teacher's suggestion that they examine the solution strategies posted on the classroom walls.

Focus on sharing ideas

Students are interested in and enthusiastic about knowing others' ideas and learning from one another. Focus your daily mathematics instruction in such a direction rather than in the direct-instruction format. A key feature of the success in this classroom was allowing students ample time (e.g., 20–30 minutes) to discuss a problem and share ideas. Krystal indicated, "This [instructional strategy] is more fun because we get to talk to each other." Many students suggested that they learned the most from the solution strategies they saw during whole-class discussion and from hearing others justify their strategies. By exploring problems and solution strategies, teachers can provide instruction that reflects state-mandated standards, promotes conceptual understanding of



mathematical ideas, and focuses on the connections between mathematical concepts.

A community of problem solvers

During the individual work time over this four-day period, students maintained a quiet focus. With eyes glued to their paper, they typically vacillated between thinking and writing. The classroom was nearly silent except for the sound of pencils and erasers moving against the paper.

Every student made an attempt at each problem during the week. After they had solved the problem individually or felt they had reached an impasse, students turned to someone sitting near them and asked for an explanation of how the classmate had solved the problem, per the discussion framework. During the course of the week, not a single student initiated a conversation by suggesting his or her solution method; all seemed eager to learn how others had solved the problem. Students indicated that they followed the discussion guidelines because they were motivated to answer the problem and felt supported by the discussion framework. Typically, the adults heard excited voices whenever a student discovered that another person had used the same strategy and found the same answer.

A distinct, observable change took place in students' behavior, how they engaged one another, how they worked, and their problem-solving discussions. Their regular summer school instructor confirmed these changes, indicating that the off-task behavior that had been present before was no longer an issue in this classroom culture.

Without a dedicated time period to share ideas, students miss the chance to learn new strategies from their peers. Moreover, a peer's explanation may be more effective than the teacher's ideas (Lampert and Cobb 2003). These fifth graders worked noticeably quicker within the framework on days three and four than on days one and two. They were excited to talk about problem-solving strategies and whether a solution was justified appropriately. To them, a solution was justified if the strategy was correctly applied and the representation of both the problem and the solution were logically related. They brought up ideas in "What about…?" and "I think…." formats, and they followed up with justification statements "… because I know…." Students also described how the classroom culture helped them feel that one another's ideas were valued.

This instructional strategy can be adapted for a typical mathematics period during the school year by using similar durations of time during the modified think-pair-share. Just imagine what you could accomplish if you were to implement these guidelines over the course of a month, a quarter, or even a year.

BIBLIOGRAPHY

Bell, Max, Jean Bell, John Bretzlauf, Amy Dillard, Robert Hartfield, Andy Isaacs, et al. *Everyday Mathematics*. Chicago: SRA/McGraw-Hill, 2004.

Lampert, Magdalene, and Paul Cobb. "Communication and Language." In *A Research Companion to Principles and Standards for School Mathematics*, edited by Jeremy Kilpatrick, W. Gary Martin, and Deborah Schifter, pp. 237–49. Reston, VA: National Council of Teachers of Mathematics: 2003.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

National Research Council (NRC). *How People Learn: Brain, Mind, Experience, and School*. Washington, DC: National Academies Press, 2000.

Pólya, George. *How to Solve It*. Princeton, NJ: Princeton University Press, 2004.

Technical Education Research Centers (TERC). Investigations in Number, Data, and Space series. Cambridge, MA: Dale Seymour Press, 2004.

Turner, Julianne C., Debra K. Meyer, Kathleen E. Cox, Candice Logan, Matthew DiCintio, and Cynthia T. Thomas. "Creating Contexts for Involvement in Mathematics." *Journal of Educational Psychology* 90, no. 4 (December 1998): 730–45.

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