## → "Building a Team of Learners" appendix

Although the original van Hiele levels were numbered 0–4, Americans started numbering the levels as 1–5 to allow for the prerecognition level 0 (Mason 1998), at which students cannot distinguish between figures.

## The van Hiele Model

Level 1 (Visualization)	Students recognize figures by appearance alone, often by comparing them with a known prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.
Level 2 (Analysis)	Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties he or she knows but not discern which properties are necessary and which are sufficient to describe the object.
Level 3 (Abstraction)	Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.
Level 4 (Deduction)	Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.
Level 5 (Rigor)	Students at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. Students at this level are able to understand the use of indirect proof and proof by contrapositive and can understand non-Euclidean systems.